Math 53, Discussions 116 and 118 **2022-10-05**

More partial derivatives

Questions

Question 1. Let $f(x,y) = ye^{xy}$. Use a linear approximation to approximate $f(0.01, 0.98)$.

Question 2. Find the tangent plane to the graph of the function $f(x,y) = 1 - \sin(x + y^2)e^{-y} - y$ at the point $(\pi, 0, f(\pi, 0)) \in \mathbb{R}^3$.

Question 3. Find the tangent plane to the surface

$$xy + yz + zx = 5$$

at the point $(1,2,1)$. Hint: You can solve for z and then compute $\partial z/\partial x$ and $\partial z/\partial y$. Or you can compute these quantities via implicit differentation without explicitly solving for z. (Soon we'll learn yet another way of solving this problem, in §14.6.)

Question 4. Check that $x = 1$ solves the equation

$$x^7 - x^6 + 2x - 2 = 0.$$

Now consider the equation

$$x^7 - 1.03x^6 + 2.06x - 2 = 0.$$

Can you linearly approximate a solution to this equation, given that $x = 1$ solved the original equation?

Below are brief answers to the worksheet exercises. If you would like a more detailed solution, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to questions

Question 1. The partial derivatives of f are

$$f_x(x,y) = y^2 e^{xy}$$
$$f_y(x,y) = e^{xy} + xye^{xy}$$

so $f_x(0,1) = 1$ and $f_y(0,1) = 1$. Hence

$$f(0.01, 0.98) \approx f(0,1) + f_x(0,1)(0.01) + f_y(0,1)(-0.02) = 1 + 0.01 - 0.02 = 0.99.$$

Question 2. The partial derivatives are

$$f_x(x,y) = -\cos(x+y^2)e^{-y}$$
$$f_y(x,y) = e^{-y}(\sin(x+y^2) - 2y\cos(x+y^2)) - 1$$

so $f_x(\pi,0) = 1$ and $f_y(\pi,0) = -1$. The tangent plane is

$$z = 1 + (x-\pi) - y.$$

Question 3. Let $F(x,y,z) = xy + yz + zx$. Then implicit differentiation yields

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{y+z}{x+y}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{x+z}{x+y}.$$

So at $(x,y,z) = (1,2,1)$ we get -1 and $-2/3$ respectively. The tangent plane is

$$z = 1 - (x-1) - \frac{2}{3}(y-2).$$

Question 4. Consider the equation

$$x^7 - ax^6 + bx - 2 = 0.$$

We can view this as defining x in terms of a, b near $(a,b,x) = (1,2,1)$. Implicit differentiation shows

$$\frac{\partial x}{\partial a} = -\frac{-x^6}{7x^6 - 6ax^5 + b}, \quad \frac{\partial x}{\partial b} = -\frac{x}{7x^6 - 6ax^5 + b}.$$

At $(a,b,x) = (1,2,1)$, these are $1/3$ and $-1/3$ respectively. So we can linearly approximate x when $a = 1.03$ and $b = 2.06$ via

$$1 + \frac{1}{3}(0.03) - \frac{1}{3}(0.06) = 0.99.$$

We have

$$(0.99)^7 - 1.03(0.99)^6 + 2.06(0.99) - 2 = 0.0017\ldots$$

which is much closer to zero than

$$1^7 - 1.03(1)^6 + 2.06(1) - 2 = 0.03.$$